Deterministic Optimization

Illustration of the optimization process

Shabbir Ahmed
Anderson-Interface Chair and Professor
School of Industrial and Systems Engineering

Summary of the optimization process

GTx

Summary of the Optimization Process

Learning goals

- Interpret solutions
- Analyze solutions
- Recognize the optimization process

Python 2.7 Code for Optimization

```
# set up optimization model
n = len(symbols)
x = Variable(n)
req_return = 0.02
ret = r.T*x
risk = quad_form(x, C)
prob = Problem(Minimize(risk),
               [sum_entries(x) = 1, ret >= req_return,
                x >= 0])

# solve problem and write solution
try:
    prob.solve()
    print "----------------------"
    print "Optimal portfolio"
    print "----------------------"
    for s in range(len(symbols)):
        print 'x[%s] = %f' %(symbols[s],x.value[s,0])
    print "----------------------"
    print 'Exp ret = %f' %(ret.value)
    print 'risk    = %f' %((risk.value)**0.5)
    print "----------------------"
except:
    print 'Error'
```

$$
\begin{aligned}
\min \quad & \mathbf{x}^\top Q \mathbf{x} \\
\text{s.t.} \quad & \mathbf{e}^\top \mathbf{x} = 1.0, \\
& \bar{\mathbf{r}}^\top \mathbf{x} \geq r_{\min}, \\
& \mathbf{x} \geq \mathbf{0}
\end{aligned}
$$

- Output:

```
----------------------
Optimal portfolio
----------------------
x[MSFT] = 0.582818
x[V] = 0.204324
x[WMT] = 0.212858
----------------------
Exp ret = 0.020000
risk    = 0.038256
----------------------
```

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Solution of Optimization Model

- Investment: MSFT: $582.8 V: $204.3 and WMT: $212.9

- The expected monthly return on our $1000 investment is $20, and its standard deviation is $38.3

- Thus if the end-of-month wealth is assumed to be Normal distributed about the mean $1020.00, the $\pm 3\sigma$ range of the end-of-month wealth is between $905.1 and $1136.4

Solution Analysis

- This involves inferring the actual decisions from the solution to the optimization model of the decision problem.
- Note that the optimization process involves various approximations and assumptions at modeling, data collection, and solution stage.
- It is crucial to check the solution for robustness and sensitivity to the underlying approximations.
- This step can be quite subjective and requires a clear understanding of the real problem and its surrounding circumstances.

Solution Analysis (contd.)

- Recall our optimal investment is:

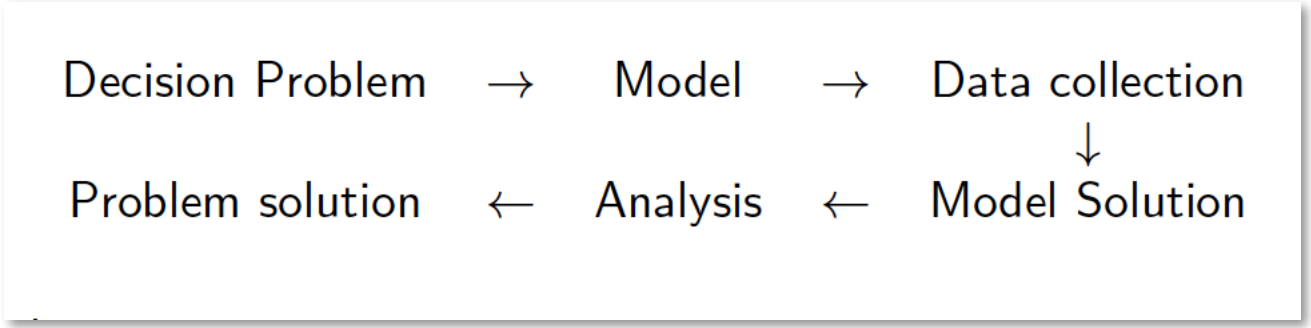
  MSFT: $582.8 V: $204.3 and WMT: $212.9
- From the last row of the data file the current prices are:

  MSFT: $74.77 V: $103.52 and WMT: $78.07
- In numbers of shares:

  MSFT: 7.79 V: 1.97 and WMT: 2.73
- Shares are only traded in whole numbers. We need to convert this solution to an implementable one.

	MSFT	V	WMT
1	44.259998	69.660004	64.839996
2	52.639999	77.580002	57.240002
3	54.349998	79.010002	58.84
4	55.48	77.550003	61.299999
5	55.09	74.489998	66.360001
6	50.880001	72.389999	66.339996
7	55.23	76.480003	68.489998
8	49.869999	77.239998	66.870003
9	53	78.940002	70.779999
10	51.169998	74.169998	73.019997
11	56.68	78.050003	72.970001
12	57.459999	80.900002	71.440002
13	57.599998	82.699997	72.120003
14	59.919998	82.510002	70.019997
15	60.259998	77.32	70.43
16	62.139999	78.019997	69.120003
17	64.650002	82.709999	66.739998
18	63.98	87.940002	70.93
19	65.860001	88.870003	72.080002
20	68.459999	91.220001	75.18
21	69.839996	95.230003	78.599998
22	68.93	93.779999	75.68
23	72.699997	99.559998	79.989998
24	74.769997	103.519997	78.07

Optimization Process



- No clear-cut recipe
- Lots of feedbacks and iterations
- Approximations and assumptions involved at each stage
- Success requires good understanding of the actual problem

Summary

- We got a sample view of the optimization process for a portfolio design example
- The optimization process involves understanding the decision problem, building a model, analyzing data, solving the model and analyzing the solution
- Very important to be aware of the assumptions made each stage