Course Syllabus

ISYE 6669

Deterministic Optimization

Spring 2021

Professors: Dr. Andy Sun and Dr. Shabbir Ahmed

Course Description

The course will teach basic concepts, models, and algorithms in linear optimization, integer optimization, and convex optimization. The first module of the course is a general overview of key concepts in optimization and associated mathematical background. The second module of the course is on linear optimization, covering modeling techniques, basic polyhedral theory, simplex method, and duality theory. The third module is on nonlinear optimization and convex conic optimization, which is a significant generalization of linear optimization. The fourth and final module is on integer optimization, which augments the previously covered optimization models with the flexibility of integer decision variables. The course blends optimization theory and computation with various applications to modern data analytics.

Prerequisite

- Linear algebra
- Multivariate Calculus
- Basic Probability
- o Familiarity with programming in Python

Course Goals

Student who take this course can expect to achieve the following goals:

- Learn modeling skills for formulating various analytics problems as linear, convex nonlinear, and integer optimization problems
- Learn basic optimization theory including duality theory and convexity theory, which will give the students a deeper understanding of not only how to formulate an optimization model, but also why.

- Learn fundamental algorithmic schemes for solving linear, nonlinear, and integer optimization problems.
- Learn computational skills for implementing and solving an optimization problem using modern optimization modeling language and solvers.

Grading Policy

- There will be one midterm quiz and one final quiz that will be graded by faculty.
 The midterm will be 35% and the final will be 40% of the overall grade.
- There will be homework assignments most weeks of the semester. Your two lowest homework grades will be dropped, and the remaining ones will add up to 25% of the course grade. Some of the assignments will be faculty graded, and others will be peer-graded (based on the median score assigned by your peer graders). You will also need to peer-grade others' homework; you will not receive a final grade for your homework submission if you do not complete your peer assessments.
- For OMS Analytics degree students, quizzes will be scaled to letter grades based on their difficulty and combined with the homework to determine an overall letter grade scale at the end of the semester.
- Grade Breakdown

| | Total | 100% |
|---|----------------|------|
| - | I IIIdi LAdili | 4070 |
| • | Final Exam | 40% |
| • | Midterm | 35% |
| • | Homework | 25% |

Homework and Quizzes Due Dates

All homework and quizzes will be due at the times in the table at the end of this syllabus. These times are subject to change so please check back often. Please convert from UTC to your local time zone using a <u>Time Zone Converter</u>.

Timing Policy

- o The Modules follow a logical sequence
- Assignments should be completed by their due dates.
- Quizzes must be completed during the time allotted on the schedule.
- You will have access to the course content for the scheduled duration of the course.

Quiz Policy

- No notes, books, or calculator/computers are allowed in the midterm and final quizzes.
- For midterm and final quizzes, you are allowed a blank sheet of paper for scratch work (OMS Analytics degree students will be proctored; you will have to show the front and back of the blank sheet while you are being proctored).

Attendance Policy

- This is a fully online course.
- Login on a regular basis to complete your work, so that you do not have to spend a lot of time reviewing and refreshing yourself regarding the content.

Plagiarism Policy

 Plagiarism is considered a serious offense. You are not allowed to copy and paste or submit materials created or published by others, as if you created the materials. All materials submitted and posted must be your own.

Student Honor Code

All OMS Analytics degree students should abide by the Georgia Tech Student Honor Code

- o Review the Georgia Tech Student Honor Code: www.honor.gatech.edu.
- Any OMS Analytics degree student suspected of behavior in violation of the Georgia Tech Honor Code will be referred to Georgia Tech's Office of Student Integrity.

Communication

- All learners should ask questions, and answer their fellow learners' questions, on the course discussion forums. Often, discussions with fellow learners are the sources of key pieces of learning.
- OMS Analytics degree students can also ask questions of the instructor and teaching assistants via Piazza. For special cases such as failed submissions due to system errors, missing grades, failed file uploads, emergencies that prevent you from submitting, personal issues, etc., a special email address will be provided in a discussion forum for you to directly contact the instructor and teaching assistants.

Netiquette

- Netiquette refers to etiquette that is used when communicating on the Internet.
 Review the Core Rules of Netiquette. When you are communicating via email, discussion forums or synchronously (real-time), please use correct spelling, punctuation and grammar consistent with the academic environment and scholarship¹.
- In Georgia Tech's MS in Analytics program, we expect all participants (learners, faculty, teaching assistants, staff) to interact respectfully. Learners who do not adhere to this guideline may be removed from the course.

¹Conner, P. (2006-2014). Ground Rules for Online Discussions, Retrieved 4/21/2014

from https://tilt.colostate.edu/teachingResources/tips/tip.cfm?tipid=128

Course Topics and Sample Pacing Schedule

• The table below contains a course topic outline and homework due dates.

| Weeks | Course Topics | Release Dates |
|--------------------|---|--|
| Week 1 | Lesson 1: Introduction to Optimization Models Lesson 2: Mathematical ingredients Lesson 3: Classification of optimization problems Module 2: Illustration of the optimization process Lesson 1: A portfolio optimization problem Lesson 2: Formulating a portfolio optimization model Lesson 3: Solving the portfolio optimization model Lesson 4: Summary of the optimization process | Jan 14, 2021 at 8: 00 a.m. Eastern |
| Week 1 Homework | Homework 1 | Jan 14 at 8: 00 a.m. Eastern – Jan 24 at 11:59 p.m. |

| | | Peer Assmt: Jan 24 at 11:59 Eastern – Jan 28 at 11:59 Eastern |
|--------------------|--|--|
| Week 2 | Module 3: Review of Mathematical Concepts Lesson 1: Linear Algebra Lesson 2: Properties of Functions Lesson 3: Properties of Sets Module 4: Convexity Lesson 1: Convex Functions Lesson 2: Convex Sets Lesson 3: Convex Optimization Problems | Jan 21, 2021 at 8: 00 a.m. Eastern |
| Week 2 Homework | Homework 2 | Jan 21 at 8: 00 a.m. Eastern – Jan 31 at 11:59 p.m. Peer Assmt: Jan 31 at 11:59 Eastern – Feb 4 at 11:59 Eastern |
| Week 3 | Lesson 1: Possible Outcomes of Optimization Lesson 2: Existence of Optimal Solutions Lesson 3: Local and Global Optimal Solutions Lesson 4: Idea of Improving Search Module 6: Optimality Certificates Lesson 1: Optimality Certificates and Relaxations Lesson 2: Lagrangian Relaxation and Duality | Jan 28, 2021 at at 8: 00 a.m. Eastern |
| Week 3 Homework | | Jan 28 at 8: 00 a.m. Eastern – Feb 7 at 11:59 p.m. Peer Assmt: Feb 7 at 11:59 Eastern – Feb 11 at 11:59 Eastern |

| Week 4 | Module 7: Unconstrained Optimization: Derivative Based • Lesson 1: Optimality Conditions • Lesson 2: Gradient Descent • Lesson 3: Newton's Method Module 8: Unconstrained Optimization: Derivative Free • Lesson 1: Methods for Univariate Functions • Lesson 2: Methods for Multivariate Function | Feb 4, 2021 at 8: 00 a.m. Eastern |
|--------------------|--|--|
| Week 4 Homework | Homework 4 | Feb 4 at 8: 00 a.m. Eastern – Feb 14 at 11:59 p.m. Peer Assmt: Feb 14 at 11:59 Eastern – Feb 18 at 11:59 Eastern |
| Week 5 | Module 9: Linear Optimization Modeling - Network Flow Problems Lesson 1: Introduction to LP Modeling Lesson 2: Optimal Transportation Problem Lesson 3: Maximum Flow Problem Lesson 4: Shortest Path Problem Module 10: Linear Optimization Modeling Electricity Market Lesson 1: How Electricity Markets Work Lesson 2: Modeling Power plant Scheduling Using LP Lesson 3: Market Clearing Mechanism Lesson 4: A Real-World Example | Feb 11, 2021 at 8: 00 a.m. Eastern |
| Week 5 Homework | Homework 5 | Feb 11 at 8: 00 a.m. Eastern – Feb 21 at 11:59 p.m. |

| | | Peer Assmt: Feb 21 at 11:59 Eastern – Feb 25 at 11:59 Eastern |
|--------------------|--|---|
| Week 6 | Module 11: Linear Optimization Modeling - Decision-Making Under Uncertainty Lesson 1: the Need to Make Decisions Under Uncertainty Lesson 2: Two-Stage Stochastic Linear Programming Lesson 3: An Example Using Stochastic LP Lesson 4: How to Solve Stochastic Programs Module 12: Linear Optimization Modeling - Handling Nonlinearity Lesson 1: The Power of Piecewise Linear Functions Lesson 2: Robust Regression Using LP Lesson 4: LP Models for Radiation Therapy Lesson 4: LP Models for Radiation Therapy | Feb 18, 2021 at 8: 00 a.m. Eastern |
| Week 6 Homework | Homework 6 | Feb 18 at 8: 00 a.m. Eastern – Feb 28, at 11:59 p.m. Peer Assmt: Feb 28 at 11:59 Eastern – Mar 4 at 11:59 Eastern |
| Week 7 | Module 13: Geometric Aspects of Linear Optimization Lesson 1: Basic Geometric Objects in LP Lesson 2: Extreme Points and Convex Hull Lesson 3: Extreme Rays and Unbounded Polyhedron Lesson 4: Representation of Polyhedrons | Feb 25, 2021 at 8: 00 a.m. Eastern |

| | Module 14: Algebraic Aspects of Linear Optimization Lesson 1: Basic Feasible Solution Lesson 2: Polyhedron in Standard Form Lesson 3: Basic Feasible Solution in Standard Form LP Lesson 4: Why We Care So Much About BFS | |
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| Week 7 Homework | Homework 7 | Feb 25 at 8: 00 a.m. Eastern – Mar 7 at 11:59 p.m. Peer Assmt: Mar 12 at 11:59 Eastern – Mar 16 at 11:59 Eastern |
| Week 8 | Lesson 1: Local Search - The General Idea Lesson 2: Local Search - Specialized to LP Lesson 3: How to Walk on the Edge Lesson 4: When to Stop and Declare Victory Module 16: Further Development of Simplex Method Lesson 1: Summarize Simplex Method Lesson 2: Handling Degeneracy Lesson 3: Phase I/Phase II Simplex Method Lesson 4: An Example | Mar 4, 2021 at 8: 00 a.m. Eastern |
| Week 8 Homework | | Mar 4 at 8: 00 a.m. Eastern – Mar 14 at 11:59 p.m. Peer Assmt: Mar 14 at 11:59 Eastern – Mar 18 at 11:59 Eastern |

| Midterm | Midterm Exam | Mar 11, 2021 at 8: 00 a.m. Eastern – Mar 18 at 11:59 p.m. Eastern |
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| Week 9 | Lesson 1: A Simple But Fundamental Trick for Deriving LP Dual Lesson 2: Relation to Lagrangian Duality Lesson 3: Weak and Strong Duality and Complementary Slackness Lesson 4: Two-Person Zero-Sum Game Module 18: Robust Optimization Lesson 1: Robustness and Its Wide Applications Lesson 2: Robust Linear Program Lesson 3: Two-Stage Robust Linear Program | Mar 11, 2021 at 8: 00 a.m. Eastern |
| Week 9 Homework | Homework 9 | Mar 11 at 8: 00 a.m. Eastern – Mar 21 at 11:59 p.m. Peer Assmt: Mar 21 at 11:59 Eastern – Mar 25 at 11:59 Eastern |
| Week 10 | Module 19: Nonlinear Optimization Modeling - Approximation and Fitting Lesson 1: Gauss, Geodesy, and Linear Least-Squares Problem Lesson 2: SVD, Matrix Approximation, and Image Compression Lesson 3: Sparsity, Regularization, and Basis Pursuit Lesson 4: Robust Approximation Module 20: Nonlinear Optimization Modeling - Statistical Estimation Lesson 1: Parametric Distribution Estimation | Mar 18, 2021 at 8: 00 a.m. Eastern |

| | Lesson 2: Nonparametric Distribution Estimation Lesson 3: Computing Probability Bounds | |
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| Week 10 Homework | Homework 10 | Mar 18 at 8: 00 a.m. Eastern – Mar 28 at 11:59 p.m. Peer Assmt: Mar 28 at 11:59 Eastern – Apr 1 at 11:59 Eastern |
| Week 11 | Module 21: Convex Conic Programming - Introduction Lesson 1: Convex Cones and Generalized Inequality Lesson 2: From LP Duality to Conic Duality Lesson 3: Second-Order Conic Programming Lesson 4: Semi-Definite Programming Module 22: Second-Order Conic Programming - Examples Lesson 1: Projection and Separation Lesson 2: Optimal Placement Problem Lesson 3: Support Vector Machine Lesson 4: Nonlinear Discrimination | Mar 25, 2021 at 8: 00 a.m. Eastern |
| Week 11 Homework | Homework 11 | Mar 25 at 8: 00 a.m. Eastern – Apr 4 at 11:59 p.m. Peer Assmt: Apr 4 at 11:59 Eastern – Apr 8 at 11:59 Eastern |
| Week 12 | Module 23: Second-Order Conic Programming - Advanced Modeling | Apr 1, 2021 at 8: 00 a.m. Eastern |

| | Lesson 1: Conic Quadratic Representable Functions and Sets Lesson 2: More "Exotic" CQr Functions Lesson 3: Robust Linear Programming Revisited Module 24: Semi-Definite Programming - Advanced Modeling Lesson 1: Eigenvalue Optimization as SDP Lesson 2: Polynomial Optimization as SDP Lesson 3: Optimal Power Flow and SDP Relaxation Lesson 4: Structural Design | |
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| Week 12 Homework | Homework 12 | Apr 1 at 8: 00 a.m. Eastern – Apr 11 at 11:59 p.m. Peer Assmt: Apr 11 at 11:59 Eastern – Apr 15 at 11:59 Eastern |
| Week 13 | Module 25: Discrete Optimization - Introduction Lesson 1: Why Discrete Variables Lesson 2: Discrete Optimization Challenges Lesson 3: Computational Complexity Module 26: Discrete Optimization - Modeling With Binary Variables 1 Lesson 1: Nonconvex Functions Lesson 2: Nonconvex Sets and Logical Relations Lesson 3: Logical Relations | Apr 8, 2021 at 8: 00 a.m. Eastern |
| Week 13 Homework | Homework 13 | Apr 8 at 8: 00 a.m. Eastern – Apr 18 at 11:59 p.m. Peer Assmt: Apr 18 at 11:59 Eastern – Apr 22 at 11:59 Eastern |

| Week 14 | Module 27: Discrete Optimization - Modeling With Binary Variables 2 • Lesson 1: Set Packing, Covering, Partitioning • Lesson 2: Graph and Network Problems Module 28: Discrete Optimization - Modeling Exercises • Lesson 1: Modeling Exercises - 1 • Lesson 2: Modeling Exercises - 2 • Lesson 3: Modeling Exercises - 3 | Apr 15, 2021 at 8: 00 a.m. Eastern |
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| Week 14 Homework | Homework 14 | Apr 15 at 8: 00 a.m. Eastern – Apr 25 at 11:59 p.m. Peer Assmt: Apr 25 at 11:59 Eastern – Apr 29 at 11:59 |
| Week 15 | Module 29: Discrete Optimization - Linear Programming Relaxation • Lesson 1: Linear Programming Relaxation • Lesson 2: Ideal Formulations Module 30: Discrete Optimization - Solution Methods • Lesson 1: Enumeration • Lesson 2: Cutting Plane Methods • Lesson 3: Branch-and-Bound Algorithm • Lesson 4: Heuristics | Apr 22, 2020 at 8: 00 a.m. Eastern |
| Week 15 Homework | Homework 15 | Apr 22 at 8: 00 a.m. Eastern – Apr 29 at 11:59 p.m. Peer Assmt: Apr 29 at 11:59 Eastern – May 3 at 11:59 |
| Einal | Final Exam | Eastern May 3, 2021 at 8:00 a.m. Fastern – May 9, 2021 at 11:59 |

Course Materials

- All content and course materials can be accessed online
- There is no textbook for this course
- Reference books:
 - R. Rardin. "Optimization in Operations Research", Prentice Hall, 1998.
 - S. Boyd and L. Vandenberghe, "Convex Optimization," Cambridge University Press, 2004. Online: https://web.stanford.edu/~boyd/cvxbook/
 - A. Ben-Tal and A. Nemirovski, "Lectures on Modern Convex Optimization," SIAM, 2001.

Technology/Software Requirements

- o Internet connection (DSL, LAN, or cable connection desirable)
- PulP optimization software (free download; see http://www.coin-or.org/PulP/ Windows version and (for Mac users) a Linux version)
- CVX in Python: CVXOPT, CVXPY software (available at http://cvxopt.org
 and https://cvxopt.org
- CVX in MATLAB: CVX software (available at http://cvxr.com/cvx/)
- Python programming language (free download; see http://www.python.org).
 Preferably use the Anaconda distribution (http://www.anaconda.com) with Python 2.7.
- Adobe Acrobat PDF reader (free download; see https://get.adobe.com/reader/)