LECTURE 5
MONTE CARLO SIMULATION BY EXCEL (PART 1)

AGENDA

- Random number generation by Excel: uniform distribution
- Examples in Excel

RANDOM NUMBER TABLE

52	06	50	88	53	30	10	47	99	37	66	91	35	32	00	84	57	07
37	63	28	02	74	35	24	03	29	60	74	85	90	73	59	55	17	60
82	57	68	28	05	94	03	11	27	79	90	87	92	41	09	25	36	77
69	02	36	49	71	99	32	10	75	21	95	90	94	38	97	71	72	49
98	94	90	36	06	78	23	67	89	85	29	21	25	73	69	34	85	76
96	52	62	87	49	56	59	23	78	71	72	90	57	01	98	57	31	95
33	69	27	21	11	60	95	89	68	48	17	89	34	09	93	50	44	51
50	33	50	95	13	44	34	62	64	39	55	29	30	64	49	44	30	16
88	32	18	50	62	57	34	56	62	31	15	40	90	34	51	95	26	14
90	30	36	24	69	82	51	74	30	35	36	85	01	55	92	64	09	85
50	48	61	18	85	23	08	54	17	12	80	69	24	84	92	16	49	59
27	88	21	62	69	64	48	31	12	73	02	68	00	16	16	46	13	85
45	14	46	32	13	49	66	62	74	41	86	98	92	98	84	54	33	40
81	02	01	78	82	74	97	37	45	31	94	99	42	49	27	64	89	42
66	83	14	74	27	76	03	33	11	97	59	81	72	00	64	61	13	52
74	05	82	82	93	09	96	33	52	78	13	06	28	30	94	23	37	39
30	34	87	01	74	11	46	82	59	94	25	34	32	23	17	01	58	73

RANDOM NUMBERS IN EXCEL

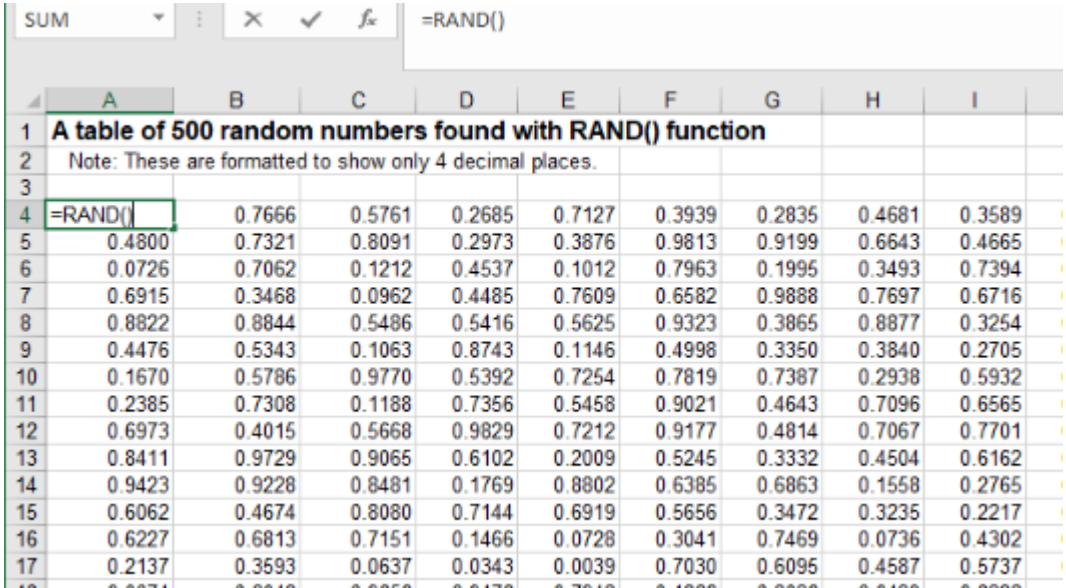
- Generation of random numbers has the following properties:
  - Uniformly distributed between 0 and 1, inclusive.
  - Probabilistically independent.
  - Easy to generate another set of random numbers for a new simulation.
  - Useful as the "building block" of a simulation.

RANDOM NUMBERS IN EXCEL: RAND FUNCTION

- Formula for generating a single random number:
  - =RAND( )
  - Parentheses (with nothing between them) are to indicate function without arguments.
- When this is copied to another set of cells, a different set of random numbers is generated.
- See Excel "Random Numbers" worksheet

EXPERIMENTING WITH RAND FUNCTION

- Enter "= RAND( )" in one cell.
- Copy it to 499 other cells.
- The numbers change whenever you change the spreadsheet.
  - You can obtain a new set of random numbers by pressing "F9".



	A	B	C	D	E	F	G	H	I
1	**A table of 500 random numbers found with RAND() function**								
2	Note: These are formatted to show only 4 decimal places.								
3									
4	=RAND()	0.7666	0.5761	0.2685	0.7127	0.3939	0.2835	0.4681	0.3589
5	0.4800	0.7321	0.8091	0.2973	0.3876	0.9813	0.9199	0.6643	0.4665
6	0.0726	0.7062	0.1212	0.4537	0.1012	0.7963	0.1995	0.3493	0.7394
7	0.6915	0.3468	0.0962	0.4485	0.7609	0.6582	0.9888	0.7697	0.6716
8	0.8822	0.8844	0.5486	0.5416	0.5625	0.9323	0.3865	0.8877	0.3254
9	0.4476	0.5343	0.1063	0.8743	0.1146	0.4998	0.3350	0.3840	0.2705
10	0.1670	0.5786	0.9770	0.5392	0.7254	0.7819	0.7387	0.2938	0.5932
11	0.2385	0.7308	0.1188	0.7356	0.5458	0.9021	0.4643	0.7096	0.6565
12	0.6973	0.4015	0.5668	0.9829	0.7212	0.9177	0.4814	0.7067	0.7701
13	0.8411	0.9729	0.9065	0.6102	0.2009	0.5245	0.3332	0.4504	0.6162
14	0.9423	0.9228	0.8481	0.1769	0.8802	0.6385	0.6863	0.1558	0.2765
15	0.6062	0.4674	0.8080	0.7144	0.6919	0.5656	0.3472	0.3235	0.2217
16	0.6227	0.6813	0.7151	0.1466	0.0728	0.3041	0.7469	0.0736	0.4302
17	0.2137	0.3593	0.0637	0.0343	0.0039	0.7030	0.6095	0.4587	0.5737

EXPERIMENTING WITH RAND FUNCTION

- Develop a frequency table to see how the 500 random number are distributed.
  - Enter a column of "bin values" (the upper ends of the categories): 0.1, 0.2, up to 0.9
  - Highlight an adjacent range one cell longer than the bin range.
  - Enter formula =FREQUENCY(data,bin) and press Ctrl-Shift-Enter.

EXPERIMENTING WITH RAND FUNCTION

- This table shows the frequencies (number of values) in each of several categories.
- The first frequency is the number of values less than or equal to 0.1.
- These were found with the FREQUENCY function (an array function).

Distribution of random numbers	
Value	Frequency
0.1	42
0.2	49
0.3	40
0.4	36
0.5	45
0.6	67
0.7	58
0.8	66
0.9	42
>0.9	55

FREEZING THE RANDOM NUMBERS

- Copy the random numbers on the original sheet to a new sheet with the Paste Special/Values option.
- The pasted copy is now frozen (does not change when F9 key is pressed).
- Freezing random numbers is sometimes useful in simulation models, but not always.
- See "Freezing Numbers" worksheet.

SIMULATING VALUES FROM PROBABILITY DISTRIBUTIONS

- RAND() generates random number in [0,1], such that the probability of generating a number within $[a, b]$ is equal to $b - a$ $(0 \leq a \leq b \leq 1)$.
- This is somewhat like drawing 00-99 out of a black box, except that we can also draw numbers like 0.5155, 0.56564, etc.
- Goals:
  - Divide [0,1] into intervals that reflect our probability distribution (i.e. random number assignment).
  - Use Excel functions to identify the random observation based on the generated random number.

EXAMPLE 1: MONTE CARLO SIMULATION REVISIT WITH EXCEL

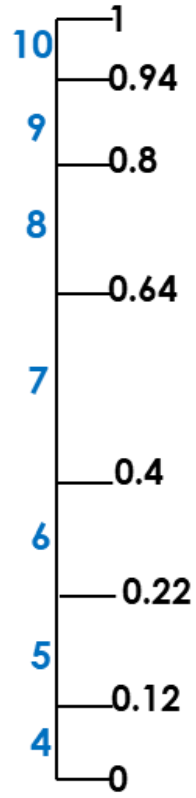
- Last class: Higgins Plumbing and Heating
- Use simulation to estimate the average weekly sales.

Heater Sales	Probability
4	0.12
5	0.10
6	0.18
7	0.24
8	0.16
9	0.14
10	0.06
Total	**1.00**

STEP 1: ASSIGN RANDOM NUMBER INTERVALS

- When we specify the random number intervals, it will seem as if the upper limit of one row overlaps with the lower limit of the next row.
- However, the probability that we get any specific number like 0.1, 0.5, etc with the RAND() function is essentially 0.
- Don't need to worry about RAND() being equal to the lower/upper limits.

	A	B	C	D	E
1	**Heater Sales**	**Probability**	**Cumulative Distribution**	**Lower limit of random number interval**	**Upper limit of random number interval**
2	4	0.12	0.12	0	0.12
3	5	0.1	0.22	0.12	0.22
4	6	0.18	0.4	0.22	0.4
5	7	0.24	0.64	0.4	0.64
6	8	0.16	0.8	0.64	0.8
7	9	0.14	0.94	0.8	0.94
8	10	0.06	1	0.94	1



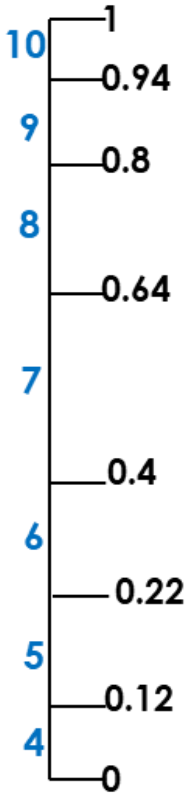
STEP 2: DETERMINE RANDOM OBSERVATION (LOOKUP)

- LOOKUP(lookup_value, lookup_vector, result_vector)
- Look for *lookup_value* in *lookup_vector* and return a value from the same position in *result_vector.*
- *lookup_value* = Our random number
- *lookup_vector* = Random number assignment
- *result_vector* = Observations
- Remark:
  1. The values in *lookup_vector* must be placed in ascending order
  2. If the LOOKUP function can't find *lookup_value*, Excel rounds DOWN to match the closest value in *lookup_vector* .
- Implication of Remark 2 → Use **Lower Limit** for *lookup_vector* .

HIGGINS - LOOKUP

- See "Higgins - Lookup" tab.

	A	B	C	D	E
1	**Heater Sales**	**Probability**	**Cumulative Distribution**	**Lower limit of random number interval**	**Upper limit of random number interval**
2	4	0.12	0.12	0	0.12
3	5	0.1	0.22	0.12	0.22
4	6	0.18	0.4	0.22	0.4
5	7	0.24	0.64	0.4	0.64
6	8	0.16	0.8	0.64	0.8
7	9	0.14	0.94	0.8	0.94
8	10	0.06	1	0.94	1
9					
10	**Random number =**	0.980523094			
11	**Simulated demand=**	=LOOKUP(B10, D2:D8, A2:A8)			



STEP 2: VLOOKUP (ALTERNATIVE)

- Alternatively, we can use the VLOOKUP function.
- VLOOKUP(lookup_value, table_array, col_index_num, range_lookup)
- Searches for a value in the leftmost column of a table, and then returns a value in the same row from a column you specify in the table.
- Remark:
  1. *Lookup_value:* Excel lookups for this in the FIRST column of the table.
  2. *Table_array* is the table of information.
  3. *Col_index_num* of x means that we return values from the x-th column of the table.
  4. *Range_lookup* is either TRUE (approximate match) or FALSE (exact match).
     - If "TRUE" or omitted, then VLOOKUP works like LOOKUP and rounds down lookup_value.
     - If "FALSE", VLOOKUP will find an exact match, or return the error value #N/A if not found.

STEP 2: VLOOKUP (ALTERNATIVE)

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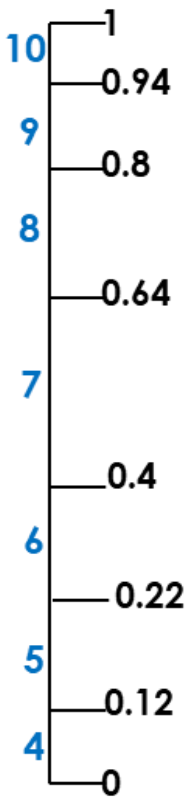
- VLOOKUP(lookup_value, table_array, col_index_num, range_lookup)
- Searches for a value in the leftmost column of a table, and then returns a value in the same row from a column you specify in the table.
- Remark:
  1. *Lookup_value:* Our random number.
  2. *Table_array:* First column of table should be the lower limit.
  3. *Col_index_num:* Column with heater sales.
  4. *Range_lookup:* TRUE

VLOOKUP FUNCTION: EXAMPLE

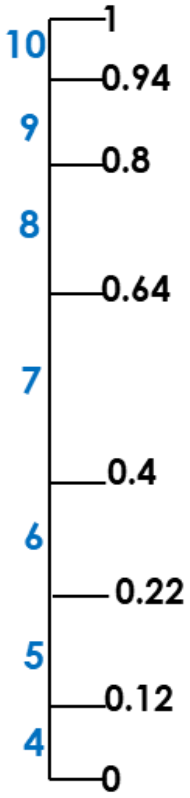
- See "Higgins - Vlookup" worksheet
- Observe that "Upper limit" = "Cumulative probability" in the last example → Don't need two different columns.

	A	B	C	D
1	**Probability**	**Lower limit of random number interval**	**Upper limit of random number interval (Cumulative Prob.)**	**Heater Sales**
2	0.12	0	0.12	4
3	0.1	0.12	0.22	5
4	0.18	0.22	0.4	6
5	0.24	0.4	0.64	7
6	0.16	0.64	0.8	8
7	0.14	0.8	0.94	9
8	0.06	0.94	1	10
9				
10	Random number =	0.739029326		
11	Simulated demand=	=VLOOKUP(B10,B2:D8,3,TRUE)		
12				



HIGGINS – SALES AND OUT-OF-STOCK

- Suppose Jerry restocks and starts each week with 8 heaters.
- Run a simulation over 100 weeks to compute the average weekly demand.
- On average, how many customers are unable to buy a heater because Jerry is out-of-stock?
- IF(logical_test, value_if_true, value_if_false)
  - Logical_test: demand <= 8
  - Value_if_true: 0 (everyone buys a heater)
  - Value_if_false: demand – 8 (# of unhappy customers)
- How can we modify the IF function if we only want to count the number of weeks that Jerry is out-of-stock?



EXAMPLE 2: ART'S NEWSSTAND

- Art sells newspaper. He cannot foretell the daily demand D (i.e. the number of newspapers that he can sell in any day).
- Each morning, Art orders q newspapers from his wholesaler at \$3 each.
- Art sells the newspaper at \$5 each. If he has unsold units at the end of the day, he receives \$0.50 per scrap (i.e. an unsold piece).
- Art wants to maximize his average daily profit. How many newspapers, q, should he order each morning?

EXAMPLE 2: ART'S NEWSSTAND

- Decision Variable: q = daily order quantity.
- Art's random daily profit:
  - If $D \geq q$, then Art earns $\$5 \times q - 3 \times q$.
  - If $D < q$, then Art earns $\$5 \times D + 0.50 \times (q - D) - 3 \times q$.
- Combining the two scenarios yields Art's random daily profit:

$$5 \times \min(q, D) + 0.5 \times \max(q - D, 0) - 3 \times q.$$

Revenue | Scrap value, if any | Purchase cost

EXAMPLE 2: ART'S NEWSSTAND

- Art wants to determine the optimal order quantity to maximize his average daily profit.
- Why focus on average profit?
- Taking expectation on Art's random daily profit yields Art's average daily profit, namely:

$$\mathrm{E}[5 \times \min(q, D) + 0.5 \times \max(q - D, 0) - 3 \times q]$$

- Maximizing Art's average daily profit is expressed by

$$\max_{q} \ \mathrm{E}[5 \times \min(q, D) + 0.5 \times \max(q - D, 0) - 3 \times q]$$

- How can we determine the optimal order quantity?

EXAMPLE 2: ART'S NEWSSTAND

- Art cannot control the daily demand.
- Hence, the daily demand D can be described by a random variable.
- An idea without computer:
  - Order 80 newspaper over the next month.
  - Compute the average (expected) daily profit when Art orders $q = 80$ newspapers.
  - Repeat for different values of q.

EXAMPLE 2: ART'S NEWSSTAND

- Alternatively, determine the probability distribution of past demand and run simulations.
- Q1: Suppose he orders 80 newspaper each morning. Can we compute his average profit?
  - Be careful! Our goal is to compute Art's average profit, NOT average demand.

EXAMPLE 2: ART'S NEWSSTAND

- Art tells us that the daily demand follows a uniform distribution between 30 pieces and 150 pieces, inclusive.
- That is, $D \sim U[30,150]$ and every integer between 30 and 150 occurs with EQUAL probability.
- When we have a uniform distribution between a and b, we can use the *RANDBETWEEN*(a,b) function as a shortcut!

EX 2, Q1: COMPUTING AVERAGE PROFIT

	A	B	C
1	Decision: Order size	80	
2	Average Profit:	111.832	
3			
4			
5			
6	Number of Trials	Newspaper Demand	Art's Profit
7	1	138	160
8	2	87	160
9	3	143	160
10	4	84	160
11	5	58	61
12	6	149	160
13	7	146	160
14	8	60	70
15	9	136	160
16	10	62	79

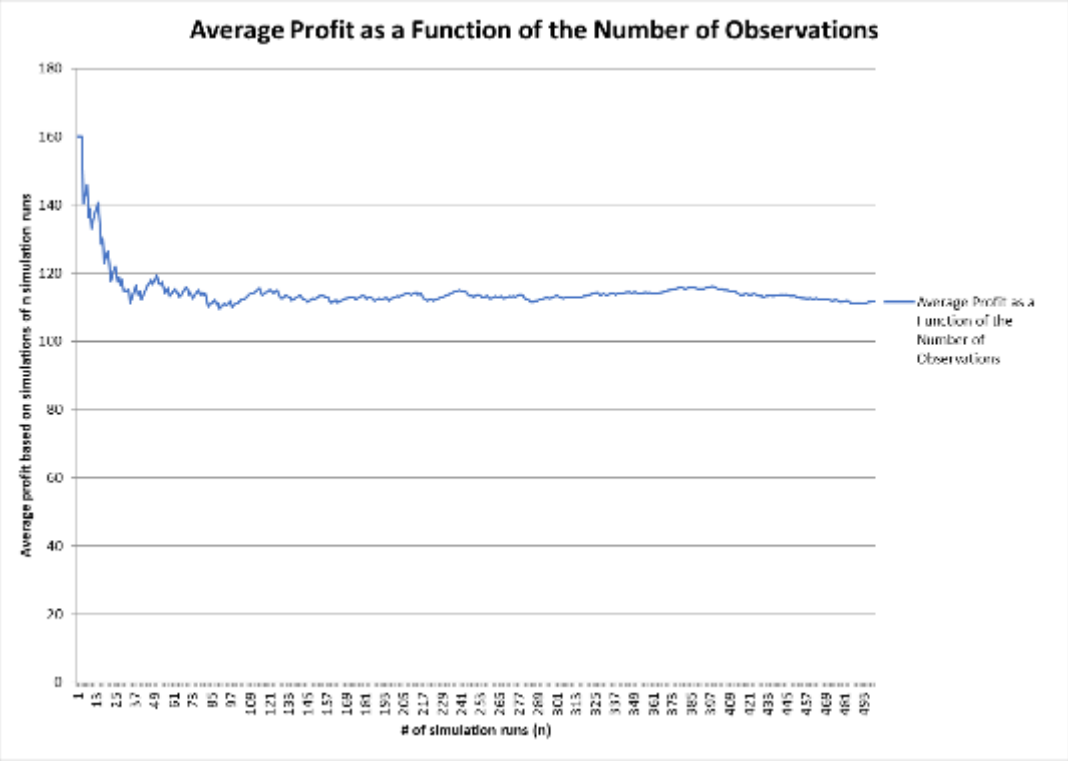
Average over profits in Col C

Profit
=5*MIN(B1,B7)
+0.5*MAX(0,B1-B7)
-3*B1

Simulated Demand
=RANDBETWEEN(30,150)

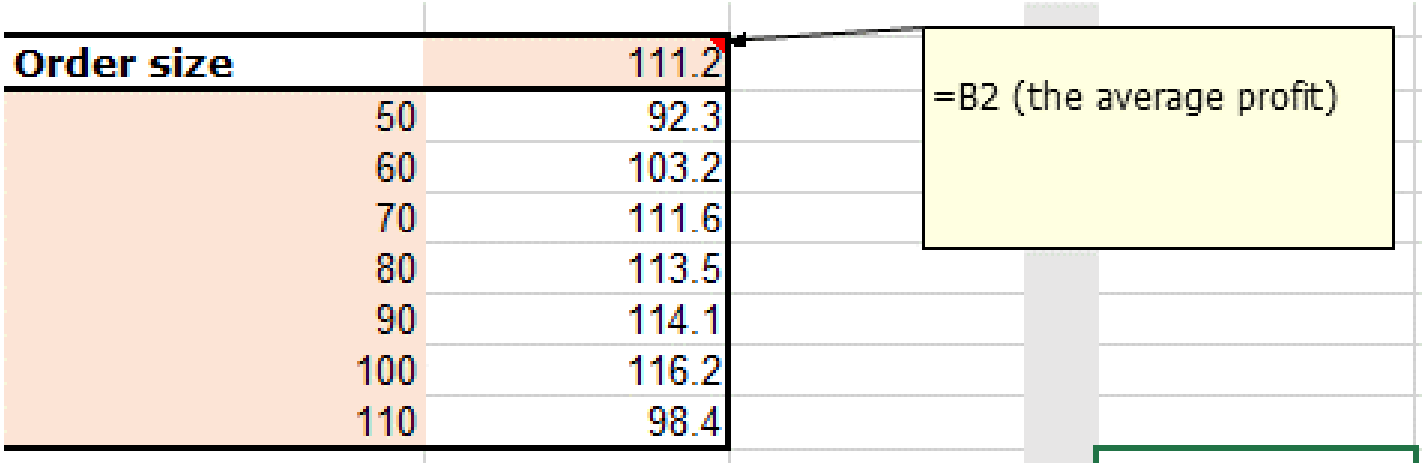
EXAMPLE 2: ART'S NEWSSTAND

- Average Profit as a Function of the Number of Observations, given that the daily order quantity is $q = 80$.



EX Q2: OPTIMAL ORDER QUANTITY

- Q2: How many newspaper should Art order daily to maximize his average profit?
- Method 1: Data table
- Method 2: Scenario Manager



SUMMARY

- Monte Carlo simulation via Excel
- Computing an optimal decision using simulation