LECTURE 6
MONTE CARLO SIMULATION BY EXCEL (PART 2)

AGENDA

- Random number generation by Excel: other distributions
- What-if analysis

MONTE CARLO SIMULATION REVISIT WITH EXCEL

- Higgins Plumbing – Demand follows a general discrete distribution (table on right)
- Art's Newsstand – Demand follows uniform distribution, $U[30,150]$
- What if demand follows the normal distribution, or other distributions that you might have heard of?

Heater Sales	Probability
4	0.12
5	0.10
6	0.18
7	0.24
8	0.16
9	0.14
10	0.06
Total	**1.00**

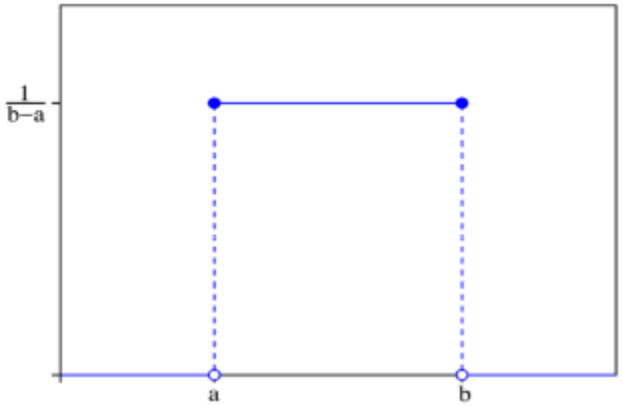
CONTINUOUS UNIFORM DISTRIBUTION

- See "a+(b-a)Rand( )" worksheet.
- If a random variable follows a continuous uniform distribution $\mathrm{U}[a, b]$, then all the values (real numbers, i.e. numbers with decimals) that lie between the lower limit a and the upper limit b are equally likely to be selected. To simulate such a random variable, we can use the formula

$$= a + (b - a) \times RAND( )$$

- E.g. If you want real numbers between 3 and 9 inclusively, use

$$= 3 + (9 - 3) * RAND( ).$$



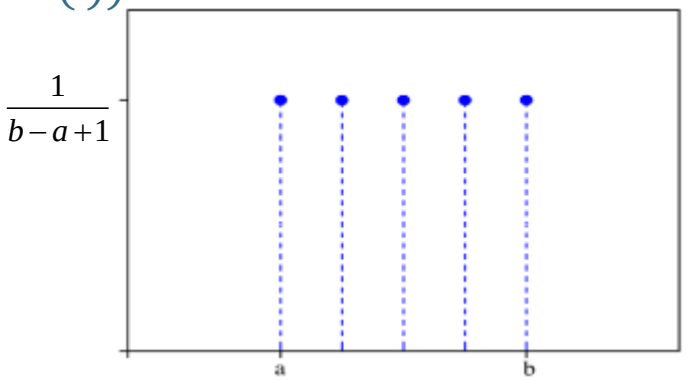
DISCRETE UNIFORM DISTRIBUTION

- See "RANDBETWEEN( )" worksheet.
- If a random variable follows a discrete uniform distribution $\mathrm{U}[a, b]$, then all the integers that lie between the lower limit a and the upper limit b are equally likely to be selected. To simulate such a random variable uses the formula:

$$= RANDBETWEEN(a, b) \quad \text{or} \quad = INT(a + (b - a + 1) * RAND\ ())$$

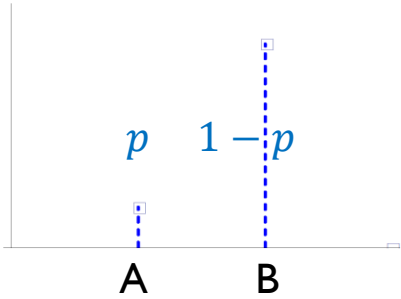
- E.g. If you want whole numbers (or called integers) between 0 and 99, use

$$= RANDBETWEEN(0{,}99) \text{ or } = INT(100 * RAND( ))$$



BERNOUILLI DISTRIBUTION

- See "Bernoulli" worksheet
- Bernoulli distribution - 2 outcomes, A and B: $Bern(p)$
  - A occurs with probability p, and B with probability $1 - p$.
  - Use the IF function: *=IF(RAND( )<p, A, B)*
- Binomial distribution - The result of n independent Bernouilli random variables.
  - Count the number of times outcome A occurs (versus B).
- E.g. A customer purchases a chocolate ice cream with 55% probability, and strawberry ice cream with probability 45%
  - Customer purchases a chocolate ice cream:
    =IF(RAND( )<0.55, "chocolate", "strawberry")
  - If 10 customers arrive, then the # of customers purchasing a chocolate ice cream follows a Binomial distribution: $Bin(n, p)$



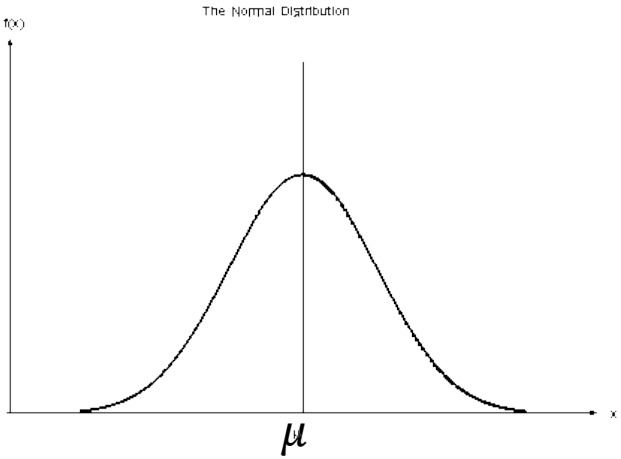
NORMAL DISTRIBUTION

- See "Normal" worksheet
- Normal Distribution: To generate random numbers from a normal distribution with mean μ and standard deviation σ, use the NORM.INV function:

$$= NORM.INV(RAND(), \mu, \sigma)$$

- Grades, height, weight, etc.
- E.g. To generate a random number from the normal distribution with mean = 10 and standard deviation = 5, use

$$= NORM.INV(RAND(\,), 10{,}5)$$



SIMULATING PROBABILITY DISTRIBUTIONS IN EXCEL

To SIMULATE	USE BULT-IN EXCEL FORMULA
Random number between 0 and 0.9999…	=RAND( )
Continuous uniform distribution between a and b	=a+(b-a)*RAND( )
Discrete uniform distribution between a and b	=INT(a+(b-a+1)*RAND ( )) or =RANDBETWEEN(a, b)
Normal distribution: Mean = μ; Standard deviation = σ	=NORM.INV(RAND(), μ, σ)
Bernoulli distribution Two outcomes: A and B, probability of A = p	=IF(RAND( )<p,A,B)
Discrete general distribution Range 1= Cell range containing lower limits of the random number intervals Range 2 = Cell range containing the variable values	=LOOKUP(RAND( ), Range 1, Range 2)

EXAMPLE 1: ABC AIRLINE

- ABC Airline provides a daily six-passenger flight to Myrtle Beach. The fare of a one-way ticket is $79, which is not refundable under no-show. The probability distribution of daily demand for such a flight and the probability distribution of no-shows (who forfeit their fares) are given in the tables on the next slide.
- The current booking policy of ABC is to oversell by three tickets. If more than 6 customers show up, then ABC must refund the fares of the extra tickets and compensate each of them $100 in cash.
- The fixed cost of each flight is $350, regardless of the number of passengers (= 0, 1, …, or 6).
- Question:
  a. Simulate ABC's profit.
  b. Investigate the outcomes where ABC oversells by 0, 1, 2, 3, 4, or 5 tickets. What are your recommendations?

EXAMPLE 1: ABC AIRLINE

- See "ABC Airline" worksheet.

Demand	Probability
5	0.05
6	0.11
7	0.20
8	0.18
9	0.16
10	0.12
11	0.10
12	0.08

No-shows	Probability
0	0.15
1	0.25
2	0.26
3	0.23
4	0.11

EXAMPLE 1: ABC AIRLINE

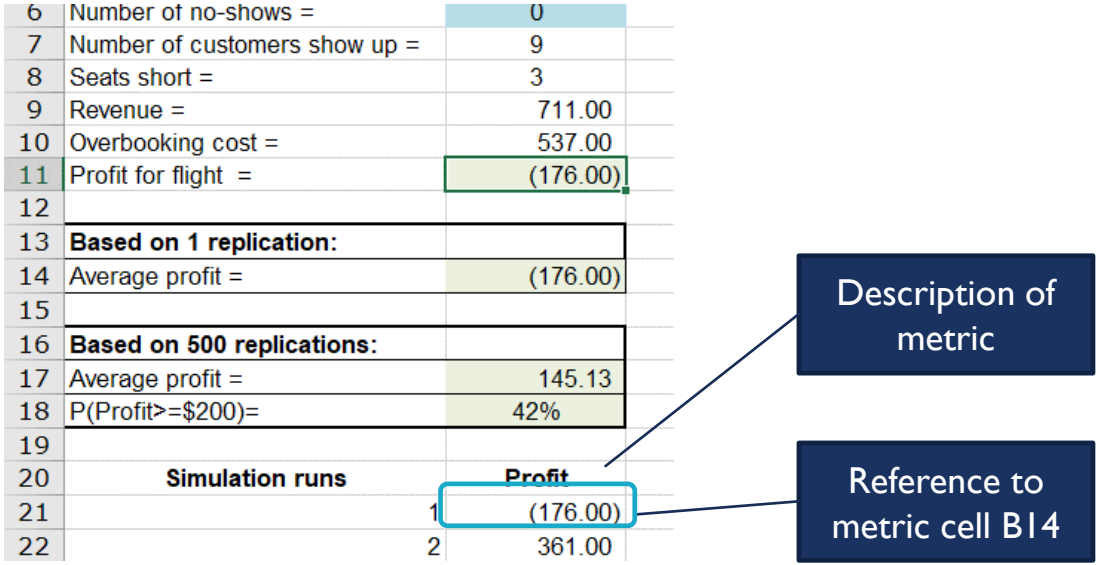
- Step 1: # of tickets sold → REVENUE
  - = minimum(demand, flight capacity + overbooking policy)
- Step 2: # of unhappy customers → COSTS
  - = maximum(0, **# of customers who show up** – flight capacity)
  - **# of customers who show up** = # of tickets sold - # of no-shows
- Step 3: Profit
  - Revenue = $79 x # of tickets sold
  - Overbooking cost = ($100+$79) x # of unhappy customers
  - Profit = revenue – overbooking cost – fixed cost

EXAMPLE 1: ABC AIRLINE

- B3: number of accepted reservations = capacity (6)+ number of overbooking (3)
- B4: actual demand =LOOKUP(RAND(),F6:F13,D6:D13)
- B5: number of reservations taken =MIN(B3,B4)
- B6: number of no shows =LOOKUP(RAND(),F16:F20,D16:D20)
- B7: number of passengers that show up =B5-B6
- B8: number of seats short =MAX(B7-F27,0)
- B9: revenue = 79 × actual reservation accepted (B5)
- B10: overbooking cost =B8*(100+79)
- B11: profit = revenue – overbooking cost – fixed cost

USE DATA TABLE TO REPLICATE MULTIPLE RUNS

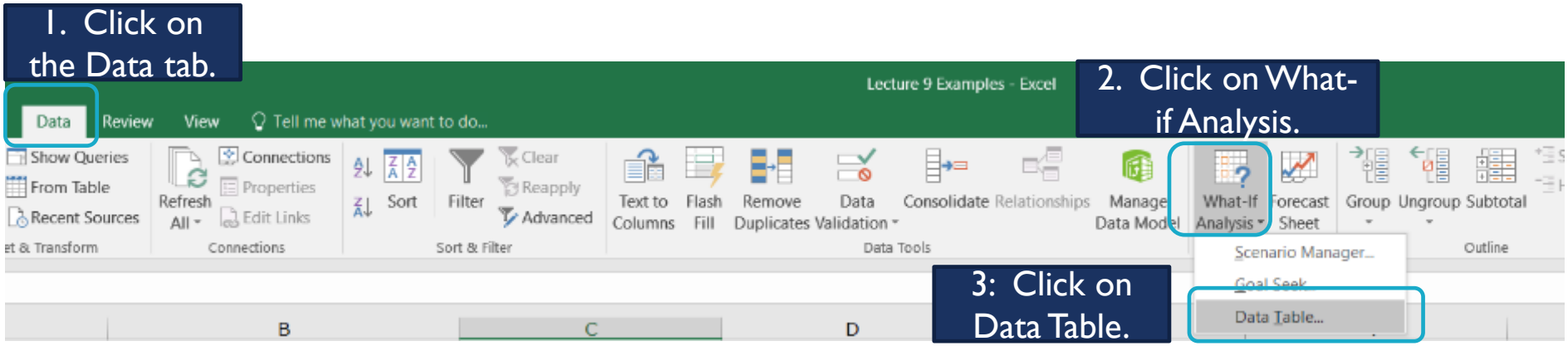
- One idea: "Press F9" (run the base model) many times, and track the results (metric).
- A better way to do this is via the Data Table function.
- **Step 1:** Create a table with two columns.
  - 1st column: Simulation runs "1, 2, 3, ..." (number of run).
  - 2nd column: Name the metric and reference to the cell containing the metric.



6	Number of no-shows =	0
7	Number of customers show up =	9
8	Seats short =	3
9	Revenue =	711.00
10	Overbooking cost =	537.00
11	Profit for flight =	(176.00)
12		
13	**Based on 1 replication:**	
14	Average profit =	(176.00)
15		
16	**Based on 500 replications:**	
17	Average profit =	145.13
18	P(Profit>=$200)=	42%
19		
20	**Simulation runs**	**Profit**
21	1	(176.00)
22	2	361.00

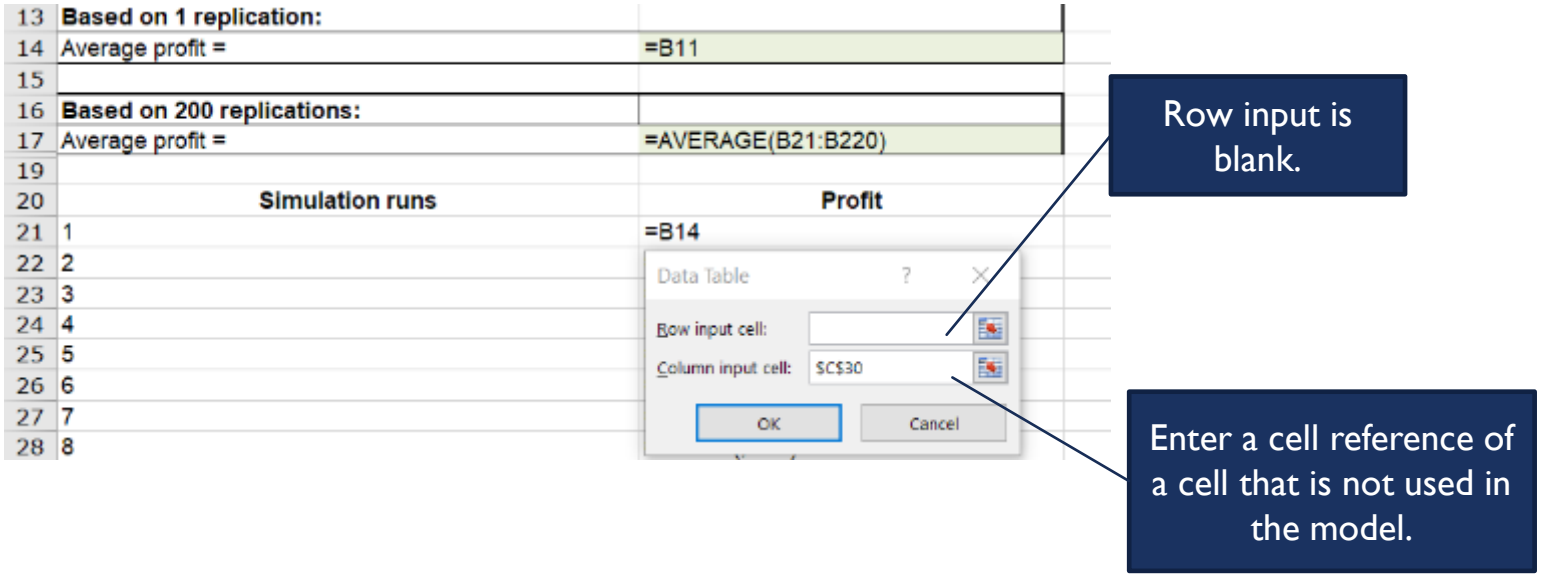
USE DATA TABLE TO REPLICATE MULTIPLE RUNS

- **Step 2:** Highlight the table with the metric reference as the top line. If we wanted to have 500 runs in the previous example, you would select cells A21:B520.
- **Step 3:** Ask Excel to fill in the table.



USE DATA TABLE TO REPLICATE MULTIPLE RUNS

- **Step 3 (cont'd):** In the Data Table options, specify a reference to an empty cell! Then, click "OK."

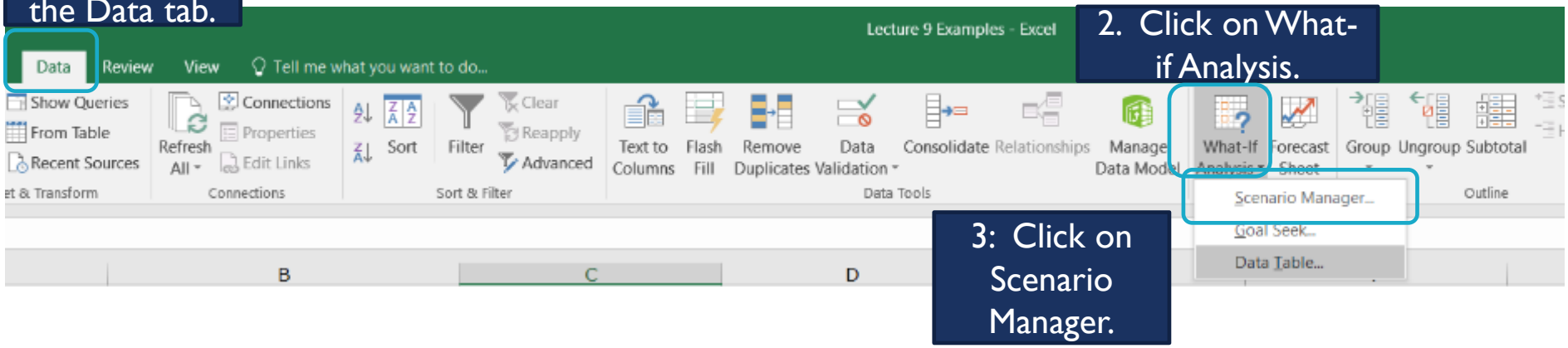


EXAMPLE 1: ABC AIRLINE

- Analysing the result
  - What is the probability that profit will be $200 or more?
  - Use of COUNTIF()
- What is the optimal overbooking policy?
  - Scenario Manager

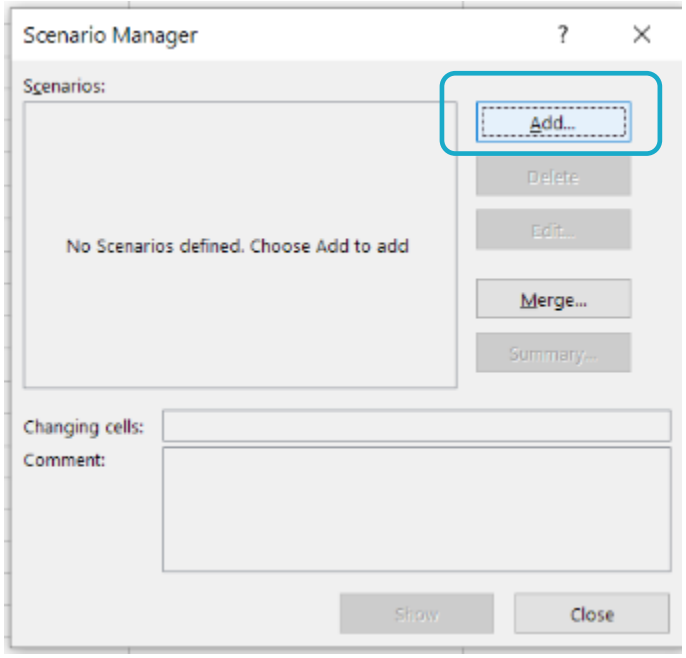
USE SCENARIO MANAGER TO INCLUDE DECISIONS

- We use "Scenario Manager" to decide the optimal "overbooking level" to maximize the expected profit

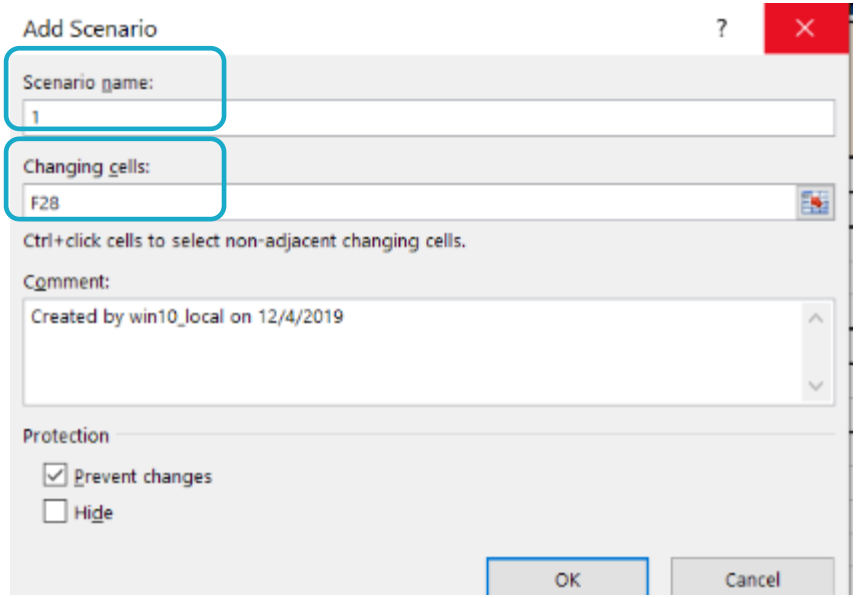


USE SCENARIO MANAGER TO INCLUDE DECISIONS

Step 1: Add Scenario.

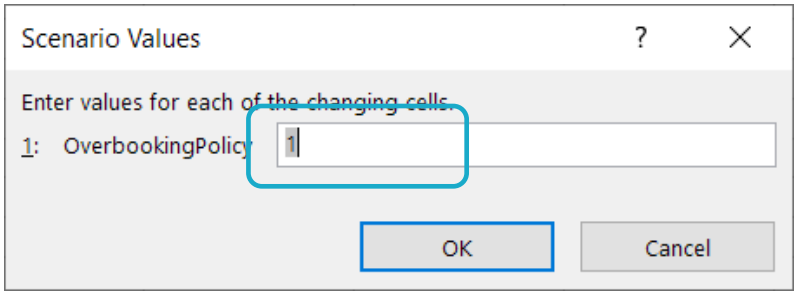


Step 2: Give "Scenario name" & specify "Changing cells".

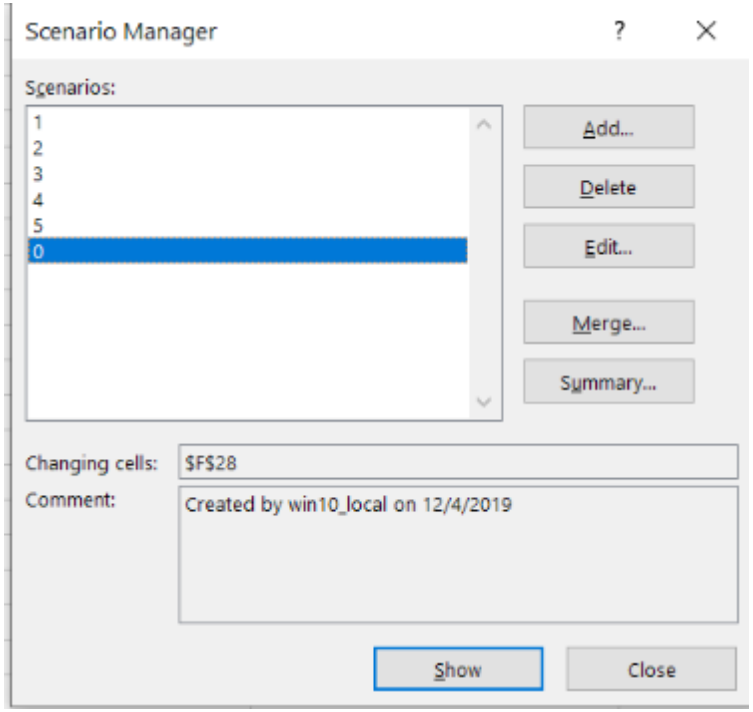


USE SCENARIO MANAGER TO INCLUDE DECISIONS

Step 3a: Enter value for each "Change cells".

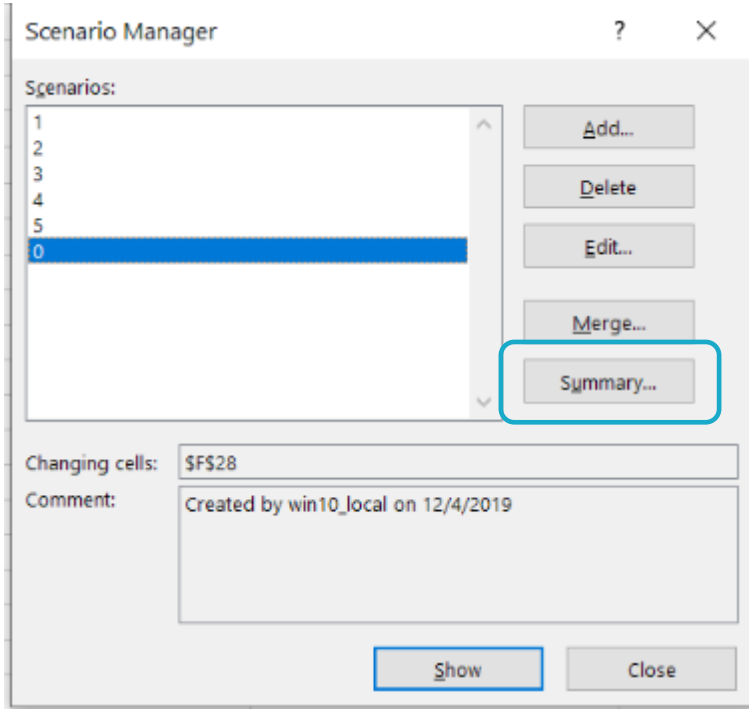


Step 3b: Add more scenarios if needed.

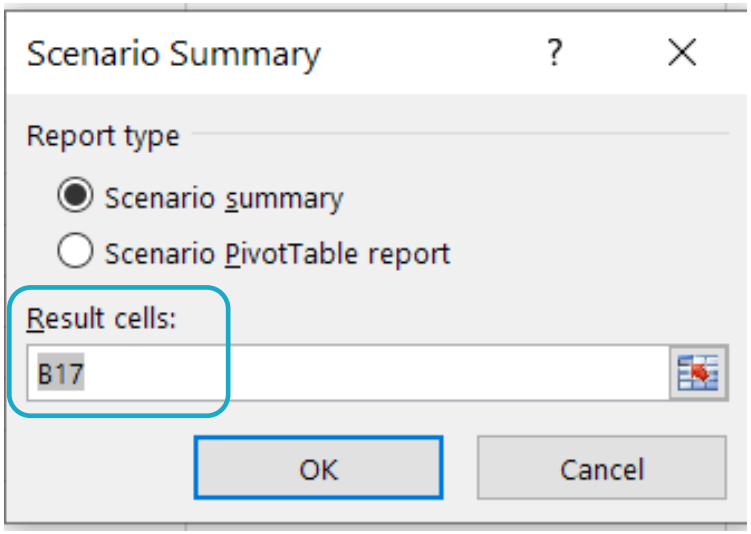


USE SCENARIO MANAGER TO INCLUDE DECISIONS

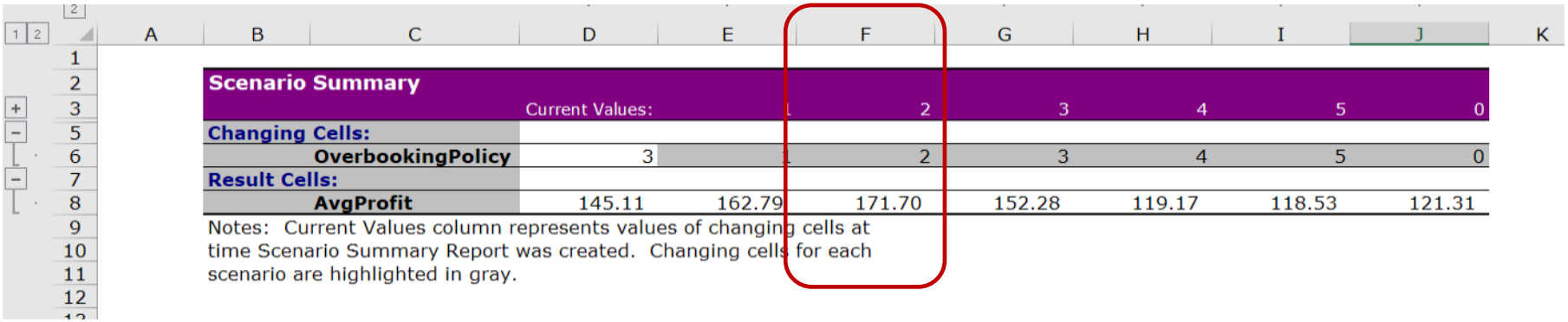
Step 4: Click "Summary to run the defined scenarios



Step 5: Specify "Result cells" to report. In this case, our "average" profit cell.



USE SCENARIO MANAGER TO INCLUDE DECISIONS



Scenario Summary	Current Values:	1	2	3	4	5	0
Changing Cells:							
OverbookingPolicy	3	1	2	3	4	5	0
Result Cells:							
AvgProfit	145.11	162.79	171.70	152.28	119.17	118.53	121.31

Notes: Current Values column represents values of changing cells at time Scenario Summary Report was created. Changing cells for each scenario are highlighted in gray.

THEORY: THE ANALYTICAL APPROACH

- Profit is a function of overbooking decision n, uncertain demand d, and no-shows s.
- $\Pi(n, d, s)$ = profit for a fixed n, realized observation of demand d, and no-shows s.
- Maximizing expected profit for different overbooking decisions, assuming independence of d, s:
  $\max_n \mathrm{E}_{d,s}[\Pi(n, d, s)|n] = \max_n \sum_d \sum_s \Pi(n, d, s) \times Prob(d) \times Prob(s)$.

Scenario manager: Compute $E[\Pi(n, d, s)|n]$ over different values of n so that we can find the optimal n.

Data table: Fix n and compute $E[\Pi(n, d, s)|n]$ by running many simulation runs

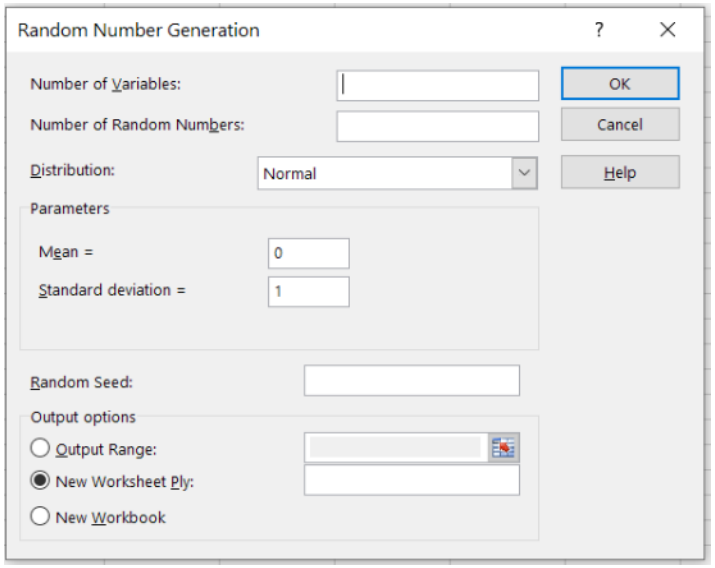
- Prob(profit ≥ \$200 | n) is found by summing the joint probability, $Prob(d) \cdot Prob(s)$, over all pairs of (d, s) that satisfies $\Pi(n, d, s)$ ≥ \$200.

DEVELOPING THE SPREADSHEET MODEL SUMMARY OF BASIC STEPS

- Use a random number to simulate a single run (or trial).
- Use the copy command or a data table (with blank column input cell) to simulate multiple runs in a single simulation (or replication).
- Calculate summary measures based on the outcomes of runs and create relevant graphs.

MORE ON RANDOM NUMBER IN EXCEL

- How to generate random numbers associated with a given distribution?
  - See "RN Generation Tool" worksheet.
  - Data →Data Analysis → Random Number Generation.



Random Number Generation for N(8,1)			
Index	R.V. #1	R. V. #2	
	7.421889	7.389393	
	8.706409	8.117273	
	6.349394	8.185846	
	8.104419	8.25665	
	7.404988	9.749513	
	7.942249	8.358876	
	7.304155	9.631206	
	7.775557	7.797627	
	7.496804	6.690887	
	7.496108	8.422126	<-- these t

SUMMARY

- Generating random numbers from different probability distributions.
- Using simulation to identify decision which optimizes some expected value function.